Analysis of Minimizing the Transportation Cost using Least Cost and Vogel's Approximation Methods

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ABSTRACT

In industries, raw material (finished products) is transported from factories to warehouses, which involves transportation cost. To reduce this cost, transportation methods are used. In Mathematics and Economics, transportation theory is the name given to the study of optimal transportation and allocation of resources. Transportation methods are strongly recommended to minimize the transportation cost. Transportation models application is very helpful to organize the process of goods distribution. This paper analyses on minimizing the transportation cost based on Least Cost and Vogel's Approximation Methods. By analyzing differences between these two methods such as number of iterations, runtime, initial solution and optimal solution, we can know which method is more effective.

KEYWORDS: *transportation cost, problem solving, computational mathematics, Vogel's approximation, minimum cost*

1. INTRODUCTION

The transportation models are primarily concerned with the optimal way in which a product produced at different factories can be transported to a number of warehouses or customers. The objective in a transportation problem is to fully satisfy the destination requirements within the production capacity constraints at the minimum possible cost. Each source is able to supply a fixed number of units of products, called availability, and each destination has a fixed demand, known as requirement. Whenever any product is produced by industry, it has to reach to its end users. Consumers may lie far away from the industry. Therefore, transportation is essential to keep the end users in access of various goods and services produced. The key factor is to decide the quantity, costs and the routes of transportation. Every source aims to minimize the cost of transportation. The objective of the system is to analyze minimizing the transportation cost using Least Cost and Vogel's Approximation Methods that are transportation methods. This system determines the amount of goods to be transported from each source to each destination that the total transportation cost is minimal.

2. RELATED WORK

Transportation problem is concerned with finding the optimal pattern of the product units' distribution from several sources to several destinations. Suppose there are *m* sources $A_1, \ldots, A_i, \ldots, A_m$ and *n* destinations $B_1, \ldots, B_j, \ldots, B_n$. The point $A_i (i = 1, \ldots, m)$ can supply a_i units, and the destination $B_j (j = 1, \ldots, n)$ requires b_j units (see Equation 1).

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \quad (1)$$

Whereby, the cost of shipping a unit from A_i to Bj, is calculated as c_{ij}. Moreover, the requirements of the destinations Bj, $j = 1, \ldots, n$, must be satisfied by the supply of available units at the points of origin Ai, $i = 1, \ldots, m$. As shown by Equation 2, if xij is the number of units that are shipped from Ai to Bj, then the problem in deciding the values of the variables xij, $i = 1, \ldots, m$ and $j = 1, \ldots, n$, should minimize the total transportation cost.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \quad x_{ij} \quad (2)$$

While,

$$\sum_{j=1}^{n} x_{ij} = a_i \; ; i = 1, \ldots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \; ; j = 1, \ldots n$$
$$x_{ij} \geq 0 \; ; i = 1, \ldots m; and\; j = 1, \ldots n$$

Since the objective function in this problem is to minimize the total transportation cost as given by Equation 3.

$$z = c_{11} \; x_{11} + c_{12} \; x_{12} + \ldots + c_{mn} \; x_{mn} \quad (3)$$

Equation 3 is a mathematical formula of a transportation problem that can adopt the linear programming (LP) technique with equality constraints. LP technique can be used in different product areas such as oil plum industry [1]. The transportation solution problem can be found with a good success in the improving the service quality of the public transport systems [2]. Also it is found in Zuhaimy Ismail at el. article [3]. As well as, the transportation solution problem is used in the electronic commerce where the area of globalization the degree of competition in the market article [4], and it can be used in a scientific fields such as the simulated data for biochemical and chemical Oxygen demands transport [5], and many other fields. Moreover, Ad-hoc networks are designed dynamically by group of mobile devices. In Ad-hoc network, nodes between source and destination act as a routers so that source node can communicate with the destination node [6].

3. THEORY BACKGROUND

The solution algorithms for transportation problems are as follows:

Step1. Formulate the problem and set up in the matrix form.

Step2. Obtain an initial basic feasible solution. This initial basic solution can be obtained by using any of the following methods:

i. North West Corner Rule
ii. Minimum Cost (Least Cost) Method
iii. Vogel's Approximation Method

The solution obtained by any of the above methods must fulfill the following conditions:

i. The solution must be feasible, i.e., it must satisfy all the supply and demand constraints. This is called RIM CONDITION.
ii. ii.The number of positive allocation must be equal to m + n – 1, where, m is number of rows and n is number of columns.

The solution that satisfies the above mentioned conditions are called a non-degenerate basic feasible solution.

Step3. Test the initial solution for optimality.

Using any of the following methods can test the optimality of obtained initial basic solution:

i. Stepping Stone Method
ii. Modified Distribution Method (MODI)

If the solution is optimal then stop, otherwise, determine a new improved solution. Repeat step 3 until the optimal solution is arrived at.

Least Cost Method (LCM)

LCM is used to obtain the initial feasible solution for the transportation problem. The allocation begins with the cell which has the minimum transportation cost. This method finds a better starting solution as it considers the cheapest transportation cost while making the allocation. Steps in Least Cost Method are:

i. Identify the cell having minimum transportation cost (C_{ij}).
ii. If there are two or more minimum costs, select the row and column corresponding to the lower numbered row.
iii. If they appear in the same row, select the lower numbered column.
iv. Choose the value of the corresponding X_{ij} as much as possible subject to the capacity and requirement constraints.
v. If demand is satisfied, delete the column.
vi. If supply is exhausted, delete the row.
vii. Repeat steps 1-6 until all restrictions are satisfied.

Vogel's Approximation Method (VAM)

VAM is an improved version of the Least Cost Method. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The penalty cost for a given supply row or demand column is defined as the difference between the lowest transportation cost and the next lowest transportation cost alternative. Steps in VAM are:

i. Determine the penalty cost for each row and column.
ii. Select the row or column with the highest penalty cost.
iii. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
iv. Repeat steps 1, 2, and 3 until all requirements have been met.

Modified Distribution Method

The Modified Distribution Method, also known as MODI method or (u-v) method provides a minimum cost solution to the transportation problem. MODI method is an improvement over the stepping stone method for testing and finding optimal solutions. It can be applied more efficiently when a large number of sources and destinations are involved. Steps in MODI method are:

i. Determine an initial basic feasible solution using any one of the three methods such as North West Corner Rule, Minimum Cost Method and Vogel’s Approximation Method.

ii. Determine the values of variables, ui and vj for each row and column using the equation ui + vj = cij (cij= transportation cost for cell ij) for occupied cells.
iii. Finding out the opportunity cost using equation kij = cij - (ui + vj) for unoccupied cells. (k_{ij} = the cost increase or decrease that would occur by allocating to a cell)
iv. If opportunity cost of all unoccupied cells are zero or greater than zero, then the optimal solution has been reached. Or else if opportunity cost is negative for one or more unoccupied cell, the solution is not optimal, transportation cost can be reduced further.
v. Select the unoccupied cell with the greatest decrease opportunity cost as the cell to be included in the next solution.
vi. Draw a closed path for the unoccupied cell selected in the previous step.
vii. Assign alternate plus and minus signs on the corner points of the closed path with a plus sign at the cell being evaluated.
viii. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

4. PROPOSED SYSTEM

This paper aims to analyze which method is more suitable for transportation problems by comparing differences such as initial solution, runtime and iteration result between Least Cost and Vogel's Approximation Methods. Then, this system also minimizes the total transportation cost. This paper studies for the system of transporting the flours. The case study of this system works with transportation problems within nine sources and eleven destinations. This system satisfies the destination requirements within the plant's capacity constraints at the minimum transportation cost. This system also determines the amount of goods to be transported from each source to each destination. The summary descriptions of the system flow of proposed system are described as follows:

i. First, we must choose names of sources and destinations and input amounts of supply and demand for a transportation problem that we want to compute.
ii. This system finds initial solution using two methods that are Least Cost and Vogel's Approximation Method.
iii. Then, this system finds the optimality of the initial solution obtained by these two methods based on Modified Distribution Method.
iv. Number of iteration, runtime, initial solution, optimal solution and amounts of goods to be transported is produced as outputs.
v. Then, by comparing the outputs of Least Cost and Vogel's Approximation Methods, we can know which method is better for minimizing the transportation cost.

Figure 1. System Flow of Proposed System

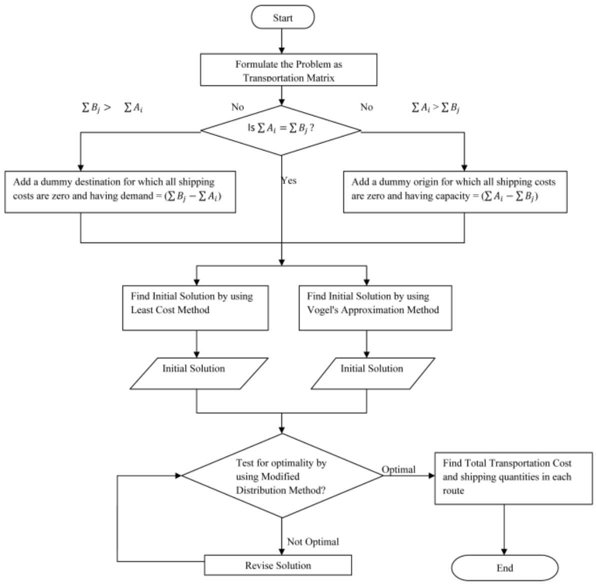

Figure 2. System Flow of Transportation Method

The descriptions of the system flow of transportation method are summarized as follow:

i. First the problem is formulated as transportation matrix.
ii. Check the problem is a balance transportation model or not?
iii. If not balance, add a dummy to either the supply or the demand to balance the transportation model.
iv. Find the initial solution of the transportation problem by using Least Cost and Vogel's Approximation Methods.
v. Check whether the solution is optimized based on Modified Distribution Method? If the solution is not optimizing, go to 5.
vi. When optimal solution is obtained, computes the total transportation cost and also shipped the respective quantity demand to its route.

5. COMPARATIVE ANALYSIS

In this part, we will compare initial solution, optimal solution, number of iteration and runtime in minimizing the total transportation cost by Least Cost and Vogel's Approximations Methods by using several of transportation problems. Vogel's Approximation method is more systematic than Least Cost method. The strength of VAM method yields the better starting basic solution than LCM because its initial solution is very near to the optimal solution. Therefore, the amount of time required to arrive at the optimum solution is greatly reduced. Then, the initial solution of VAM is less than that of LCM in most of problems. Thus, the execution time of VAM is faster than LCM in solving the transportation problems. Below, we could see the results represented with the line graph and the analysis table result to compare the execution time and number of iterations of Least Cost and Vogel's Approximation Methods with different transportation problems.

Execution Time (in milliseconds)

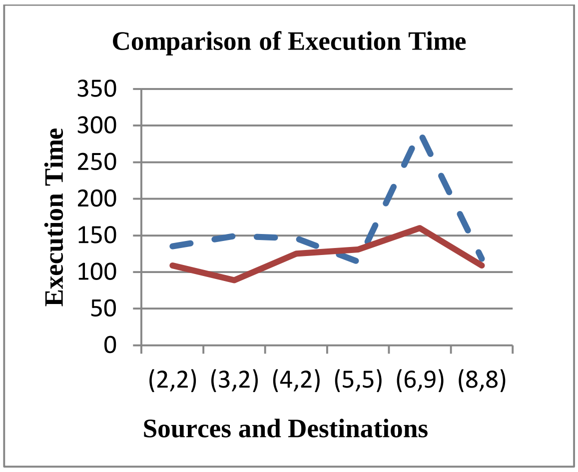

Figure 3. Line Graph (Execution Time) of Least Cost and Vogel's Approximation Methods

In figure 3, (2, 2) means the transportation problem consists of two sources and two destinations. (3, 2) means the transportation problem consists of three sources and two destinations and (4, 2) and so on. The numbers 0, 50, 100 and etc. are execution time in milliseconds to solve the problem. From the above line graph, what we can see clearly from it is that the execution time of Vogel's Approximation Method is obviously fewer than Least Cost Method.

Number of Iterations

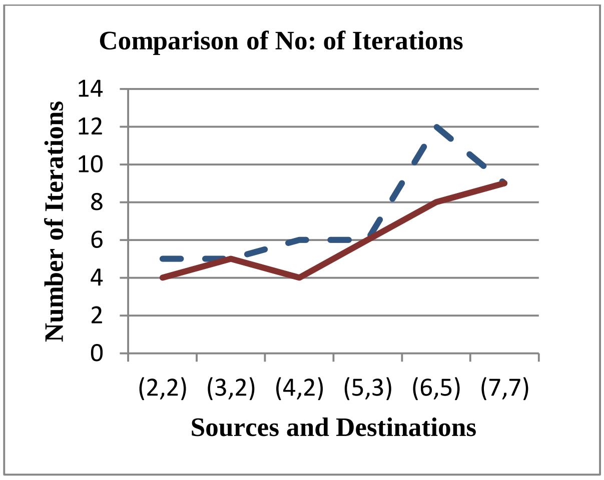

Figure 4. Line Graph (Number of Iterations) of Least Cost and Vogel's Approximation Methods

Analysis Table Result of Least Cost and Vogel's Approximation Methods

			Least Cost Method				Vogel's Approximation Method			
Supply Cities	Demand Cities	Total Quantities to be transported (kg)	No: of Iterations (Steps)	Runtime (ms)	Initial Solution (ks)	Optimal Solution (ks)	No: of Iterations (Steps)	Runtime (ms)	Initial Solution (ks)	Optimal Solution (ks)
Yangon	Pyay	600	6	82	35000	30000	5	75	30000	30000
Naypyitaw	Bago	1200	6	112	65000	60000	5	102	60000	60000
		2300	5	175	137500	137500	5	81	137500	137500
		3600	6	97	190000	180000	5	100	180000	180000
		8000	6	107	445000	400000	5	109	400000	400000
		15000	5	101	850000	850000	5	79	850000	850000

Figure 5. Analysis Table Result of Least Cost and Vogel's Approximation Methods

In the figure 4, (2, 2) means the transportation problem consists of two sources and two destinations. (3, 2) means the transportation problem consists of three sources and two destinations and (4, 2) and so on. The number 0, 2, 4, 6 and etc. are total number of iterations (steps) to get the initial solution and optimal solution. From the above line graph, the number of iterations of Vogel's Approximation Method is fewer than Least Cost Method in most problems. In some problems, number of iterations of these two methods is the same.

In figure 5, transportation problem consisting of 2 sources (supply cities) and 2 destinations (demand cities) is analyzed. Runtimes of VAM are faster than that of LCM in most transportation problems. Especially among them, runtime of LCM is faster than that of VAM in only two problems. And numbers of iteration and initial solutions of VAM are less than that of LCM in some problems. So, VAM method greatly reduces the amount of time required to reach the optimal solution. Then, there is also transportation problems that are initial solutions of LCM are the same with the initial solutions of VAM. Moreover, the optimal solutions and numbers of iterations are also the same because those problems aren't satisfy the equation (m+n-1). But runtimes of VAM are faster than that of LCM in those problems. So, as in above table result, VAM method is better than LCM method to find the initial solution for transportation problems.

6. CONCLUSION

The transportation cost is an important element of the total cost structure for any business. In this system, transportation methods are used to reduce the transportation cost. Comparisons between Vogel's Approximation Method and Least Cost Method are done in this system. As a result of analysis, we could see Vogel's Approximation Method is better than Least Cost Method in minimizing the total transportation cost. Vogel's Approximation method is a heuristics method and it is always preferred because it gives an initial solution which is nearer to an optimal solution or is the optimal solution itself. Then, Vogel's Approximation Method (VAM) obtained the optimal solution or the closest to optimal solution with a minimum computation time. Therefore, VAM method is also easily applied to find the initial basic feasible solution for the balanced and unbalanced transportation problems.

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